THE MATHEMATICS AT THE EDGE OF THE RATIONAL UNIVERSE

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These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

PRELUDE

Mathematics is the art of story-telling. Nobody has ever seen a perfectly round circle or an infinitely long line of zero width. They're pure figments of the mathematical imagination. As for imaginary square roots of −1, ideal points where parallel lines meet, and 17-dimensional space …! What fantasies can be dreamt up by the fertile mind of a mathematician!

Stories, parables, fables, myths and legends can carry profound truths that have a powerful impact on the lives we lead. Mathematical stories are no exception. This gossamer web we mathematicians spin might be pure fancy. But it's the best tool we have to understand and predict the material universe. And it reaches far beyond.

In this book we'll go on a journey to the edge of the rational universe. Our motivation will be that of an explorer. We simply want to know what's out there. Whether any practical use can be made of what we find there is not our prime concern. This book is not written for the practitioner in logic or mathematics or computing science.

Having said that let me add that the inspiration for the book came from having to teach this material to embryonic mathematicians and computer scientists in several courses at Macquarie University. I began to realise that, stripped of some of the formal technicalities, much of the material I had taught to third year students,

to honours students and even to postgraduate students could be made accessible to a wider audience.

Material which had hitherto remained locked up in courses with such intimidating names as Advanced Algebra, Axiomatic Set Theory and Theory of Computation is too fascinating to leave there. All it needs is a little less emphasis on symbolic formality and a little more imaginative presentation.

That's not to say that having read this book you'll be on a par with the students who graduate from my courses. I like to think that what I've done is to build a road into a national park that has hitherto only been accessible on foot.

I taught this material many years ago as a continuing education course for mathematical laymen (and laywomen) at Macquarie University. I even wrote an earlier, and much less complete, version of this book to give out to the students. And there it lay.

More recently, many years later, I received an email out from one of those students. She had attended the course with her father and said how much she'd enjoyed it. In fact she said, "I was thinking last night, it's still the best maths class I ever took, and one of the most fun things I ever did with my Dad." I thought it went well, but surely that must be an exaggeration! However, it inspired me to dust off those old notes and fashion them into this book.

I'm certainly not the first to have attempted to bring deep ideas of logic and mathematics to a wider

audience. Lewis Carroll was one of the first in *Alice Adventures in Wonderland* – a book which delightfully introduces many ideas of logic. I have also been influenced by Abbott's *Flatland* and the writings of Martin Gardener and Douglas Hofstadter.

The final chapter goes beyond transcendental mathematics to consider the philosophical/theological question of the existence of something beyond the material world and proofs of the existence of God. This isn't a technical discussion of epistemology but rather a drawing together of some of the ideas in the earlier chapters.

After each chapter there's a little treat – a story, poem or joke, reflecting the ideas developed in that chapter. These may or may not aid the understanding of the chapter but at least they provide some breathing space before the next one and hopefully maintain the whimsical frame of mind in which this material can best be appreciated.

This book isn't for everybody. Is it for you? Here's a check list. If you can answer "yes" to some or all of them then go ahead and read this book.

(1) Are you intrigued by the logical reflexiveness of the sentence "this sentence is false?

(2) Have you read and enjoyed *Alice's Adventures in Wonderland*?

(3) Can you cope with the symbols in the following?
Let P denote a computer program and let D denote some data on which it acts. Suppose we denote the output by P→D.

So if P is a program for duplicating data then:
$$P→D = DD.$$

And if such a program is given its own description to duplicate, we have the equation P→P = PP.

(4) Would it interest you if one could prove the existence of God?

I dedicate this book to Rachel Chalmers, whose email many years after she attended one of my courses encouraged me to write up those notes for publication. Rachel, I can barely remember who you were, but if it hadn't been for your chance email this book would probably never have been finished.

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